CSE150A/250A HW3 Discussion

Harshvardhan

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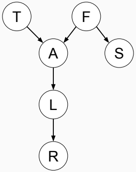
UCSD

3.1 Variable Elimination v/s Enumeration

Consider the following belief network with binary random variables.



We want to compute $P(T = 0|S = 1, R = 1)$ and $P(T = 1|S = 1, R = 1)$.

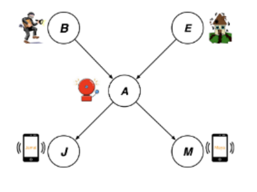
(a) Perform Variable Elimination

(b) Count the number of operations (additions, multiplications, divisions) performed during VE.

(c) Count the number of operations (additions, multiplications, divisions) performed by enumeration algorithm.

3.1 Example from class

Consider the example from class :



$$P[J] = \sum_{A,M,B,E} P[J|A]P[M|A]P[B]P[A|B,E]P[E]$$

(a) Variable Elimination Factors : $f_1(A, B), f_2(A), f_3(A)$, Elimination order : E, B, M, A

(b) Consider the factor $f_1(A, B) = \sum_E P[A|B, E]P[E]$. For each value of A, B, we need 2 multiplications and 1 addition.

(c) Enumeration : Plug in values for all random variables A, B, E, M. For each instantiation, need 4 multiplications.

Variable Elimination

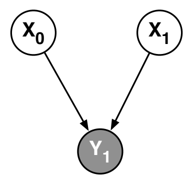
(a) Given CPTs as Factor tables and Variable Elimination Order. Perform Variable Elimination step-by-step, showing new factors created and their factor tables. Your answer should be accurate to atleast 4 decimal places. (**Suggested:** Use calculator.)

(b) Fill the table.

(c) Fill the table with a brief explanation of how you got to that number. Don't need to run the enumeration algorithm, only compute the number of operations.

Note: Conditional probabilities are stored with redundancy. Therefore, no addition or subtraction operations are required to compute $P(A = 0|T = 1, F = 1)$ based on $P(A = 1|T = 1, F = 1)$ or vice versa.

3.2 Inference in a Chain



To compute the posterior probability $P(X_1|Y_1)$, we can use Bayes rule:

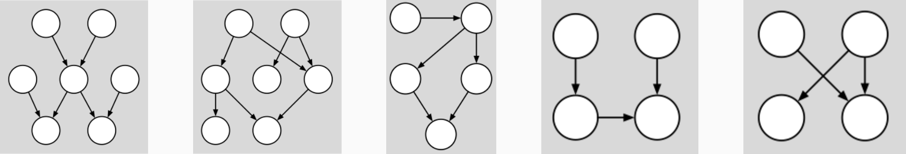
$$P(X_1|Y_1) \;=\; \frac{P(Y_1|X_1)\,P(X_1)}{P(Y_1)}$$

(a) Show how to compute the conditional probability $P(Y_1|X_1)$ that appears in the numerator of Bayes rule from the CPTs of the belief network. (2 pts)

(b) Show how to compute the marginal probability $P(Y_1)$ that appears in the denominator of Bayes rule from the CPTs of the belief network. (2 pts)

3.3 Node clustering and polytrees

In the figure below, *circle* the DAGs that are polytrees. In the other DAGs, shade **two** nodes that could be *clustered* so that the resulting DAG is a polytree.

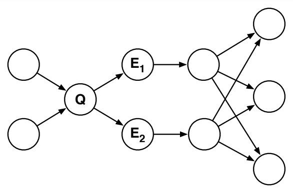


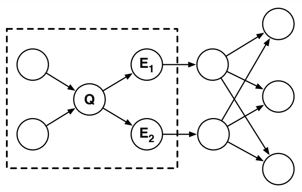
Polytree : A polytree is a singly connected belief network: between any two nodes there is at most one path.

Node Clustering : When we cluster 2 or more merge nodes in the DAG, they become a single merged node. Parents of merged node = all parents of clustered nodes. Children of merged node = all children of clustered nodes.

3.4 Cutsets and polytrees

Consider the query $P(Q|E_1, E_2)$ in the non-trivial example shown below:





In this belief network, the posterior probability $P(Q|E_1, E_2)$ can be correctly computed by running the polytree algorithm on the subgraph of nodes that are enclosed by the dotted rectangle.

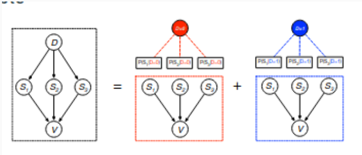
In this example, the evidence nodes *d*-separate the query node from the loopy parts of the network. Thus for this inference the polytree algorithm would terminate before encountering any loops.

3.4 Cutsets and Polytrees

For each of the five loopy belief networks in 3.4, consider how to compute the posterior probability $P(Q|E_1, E_2)$.

If the inference can be performed by running the polytree algorithm on a subgraph, enclose this subgraph by a dotted line as shown on the previous page. (The subgraph should be a polytree.) **Recall definition of a polytree.**

On the other hand, if the inference cannot be performed in this way, shade **one** node in the belief network that can be instantiated to induce a polytree by the method of cutset conditioning.



Instantiating a node breaks all edges between the node and it's children. Exact inference on the instantiated node. After instantiation, we should be able to use polytree algorithm on the children.

3.5 Even more Inference

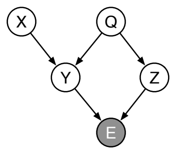
Show how to perform the desired inference in each of the belief networks shown below. Justify briefly each step in your calculations.

(a) **Markov blanket (4 pts)**
Show how to compute the posterior probability $P(B|A, C, D)$ in terms of the CPTs of this belief network—namely, $P(A)$, $P(B|A)$, $P(C)$, and $P(D|B, C)$.

(b) **Conditional independence (1 pt)**
This belief network has conditional probability tables for $P(F|A)$ and $P(E|C)$ in addition to those of the previous problem. Assuming that all these tables are given, show how to compute the posterior probability $P(B|A, C, D, E, F)$ in terms of these additional CPTs and your answer to part (a).

(c) **More conditional independence (2 pt)**
Assuming that all the conditional probability tables in this belief network are given, show how to compute the posterior probability $P(B, E, F|A, C, D)$. Express your answer in terms of the CPTs of the network, as well as your earlier answers for parts (a) and (b).

(a) **Single node of evidence (3 pts)**



Suppose that T samples $\{q_t, x_t, y_t, z_t\}_{t=1}^{T}$ are drawn from the CPTs of the belief network shown above (with fixed evidence $E = e$). Show how to estimate $P(Q{=}q|E{=}e)$ from these samples using the method of likelihood weighting. Express your answer in terms of sums over indicator functions, such as:

$$I(q, q') = \begin{cases} 1 & \text{if} \quad q = q' \\ 0 & \text{otherwise} \end{cases}$$

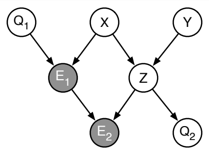
In addition, all probabilities in your answer should be expressed in terms of CPTs of the belief network (i.e., probabilities that do not require any additional computation).

Recall: Likelihood can be estimated as a ratio of counts. For likelihood weighting, we need to weigh the counts by appropriate conditional probabilities. All samples generated given evidence. Use the network to see which probabilities, we have to estimate.

(b) **Multiple nodes of evidence (4 pts)**
Suppose that T samples $\{q_{1t}, q_{2t}, x_t, y_t, z_t\}_{t=1}^{T}$ are drawn from the CPTs of the network shown below (with fixed evidence $E_1 = e_1$ and $E_2 = e_2$). Show how to estimate

$$P(Q_1 = q_1, Q_2 = q_2 | E_1 = e_1, E_2 = e_2)$$

from these samples using the method of likelihood weighting. Express your answer in terms of indicator functions and CPTs of the belief network.



How does evidence from multiple nodes interact with each other?

Thank You.